CALCULATION OF COMPRESSION SHOCKS AT LOW PRESSURE

V. F. Stepanchuk and G. A. Saltanov

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A method of calculating compression shocks at low pressure is given, together with a nomogram for water vapor which permits rapid and quite accurate calculation of the basic parameters of the shock.

In [1] a general method was proposed for calculating compression shocks in a flow of supercooled vapor. The main assumptions on which this method rests are as follows: As a rule, in the flow of moist vapor, oblique shocks occur; the shock is followed by a fan of expansion waves, whose first characteristic coincides with the shock front; the velocity of sound in the low humidity region is equal, with sufficient accuracy, to the velocity of sound in the dry saturated vapor, which in turn is given by $a = \sqrt{kRT}$; the equation $pV = RT$ is applicable to the supercooled vapor ahead of the shock and to the saturated vapor behind the shock.

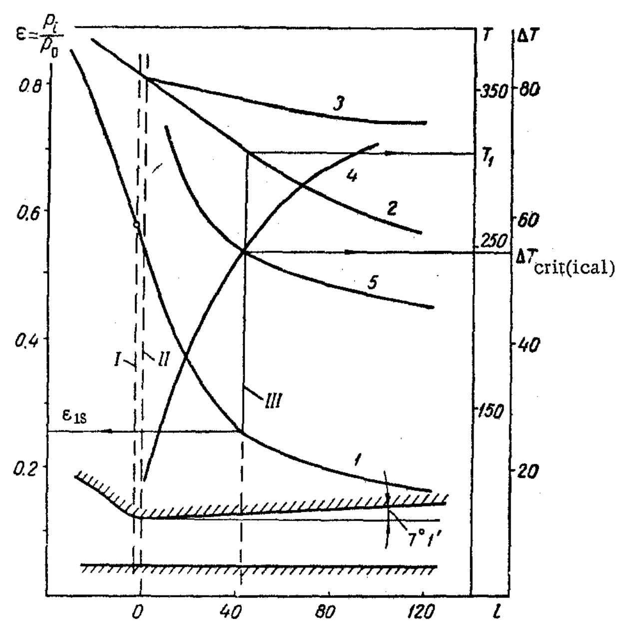

Fig. 1. Calculation of location of shock and of flow parameters ahead of it: 1) static pressure (ε); 2) static temperature of flow; 3) saturation temperature; 4) actual supercooling; 5) critical supercooling according to (1); I—section at which flow intersects upper boundary curve; II—throat; III—section at which shock occurs.

In the reference cited the following assumption, which considerably simplifies the calculation, is made: at low pressure (for water vapor static pressure ahead of the shock $p < 0.5 \times 10^5$ n/m^2) it may be assumed that the saturation temperatures ahead of and behind the shock are equal. This also holds for the latent heat of vaporization r. In fact, for water vapor with $p_1 = 0.05 \times 10^5$ n/m^2 and $p_2 = 0.1 \times 10^5$ n/m^2 ($p_2/p_1 < 2$ for oblique shocks, as is shown by experimental data and calculation) $T'_2/T'_1 = 1.04$, i.e., with respect to the saturation temperatures it may be assumed that $T_2^!/T_1^!$.

In calculating compression shocks the given quantities are p_0, T_0 and the nozzle geometry. Therefore the problem is to determine the location of the shock. The problem is solved either on the basis of the kinetics of phase transition in a flow of supercooled vapor [2], which is very difficult and requires a large volume of calculations, or on on the basis of empirical relations.

A possible empirical formula for water vapor [3] is

$$\Delta T_{\text{crit(ical)}} = b\,(1/\tau)^{0.2}. \qquad (1)$$

The coefficient b takes values from 8.8 to 9.6. Calculations based on this formula are carried out in the following order. The static pressure and temperature distribution along the nozzle is constructed starting from a value of the adiabatic exponent $k = 1.3$, then from the section at which the vapor intersects the upper boundary curve, the dependence of saturation temperature on static pressure is constructed. The difference between the saturation temperature and the static temperature gives the actual supercooling of the flow at each section of the nozzle.

The time for the flow to travel along the nozzle from the section where the vapor intersects the upper boundary curve to any given section is easily determined:

$$\tau = \int_{l'}^{l} dl/c.$$

Thus, the critical supercooling can be found at each section beyond the point at which the flow intersects the upper boundary curve. The point of intersection of the curve of critical supercooling and the curve of actual supercooling of the flow defines the location of the shock. Therefore the flow parameters p_1, T_1, c_1 ahead of the shock are known (Fig. 1).

Further calculation of the shock may be based on the equations of gas dynamics. In terms of the assumptions made above, these equation may be written as follows:

1. Equation of continuity

$$x_2 = \frac{T_1' - \Delta T}{T_1'}\,\frac{p_2}{p_1}\,\frac{M_2 \sin\beta_2}{M_1 \sin\beta_1}, \qquad (2)$$

where $M_1 = c_1/\sqrt{kRT_1}$, $M_2 = c_2/\sqrt{kRT_1}$ are dimensionless flow velocities.

2. Equation of momentum normal to the shock front

$$M_2 \sin\beta_2 = M_1 \sin\beta_1 - \frac{1}{kM_1 \sin\beta_1}\left(\frac{p_2}{p_1} - 1\right).$$

Since the first characteristic of the fan of expansion waves coincides with the shock front and the velocity of sound $a_2 = \sqrt{kRT_2}$, we obtain

$$M_2 \sin\beta_2 = \sqrt{T_1' / (T_1' - \Delta T)},$$

and therefore

$$\sqrt{\frac{T_1'}{T_1' - \Delta T}} = M_1 \sin\beta_1 - \frac{1}{kM_1 \sin\beta_1}\left(\frac{p_2}{p_1} - 1\right). \quad (3)$$

3. The equation of momentum along the shock front

$$M_1 \cos\beta_1 = M_2 \cos\beta_2. \quad (3')$$

4. Together with (3'), the energy equation gives

$$i_1 + \frac{kR}{2}(T_1' - \Delta T) M_1^2 \sin^2\beta_1 = i_2 + \frac{kR}{2}(T_1' - \Delta T) M_2^2 \sin^2\beta_2$$

or

$$i_1'' - c_p \Delta T + \frac{kR}{2}(T_1' - \Delta T) M_1^2 \sin^2\beta_1 = i_2'' - (1 - x) r_2 + \frac{kR}{2}(T_1' - \Delta T) M_2^2 \sin^2\beta_2.$$

Taking into account that $r_2 = r_1$, and $i_2'' = i_1''$, we obtain

$$\frac{kR}{2}(T_1' - \Delta T)(M_1^2 \sin^2\beta_1 - M_2^2 \sin^2\beta_2) = c_p \Delta T - (1 - x) r. \quad (4)$$

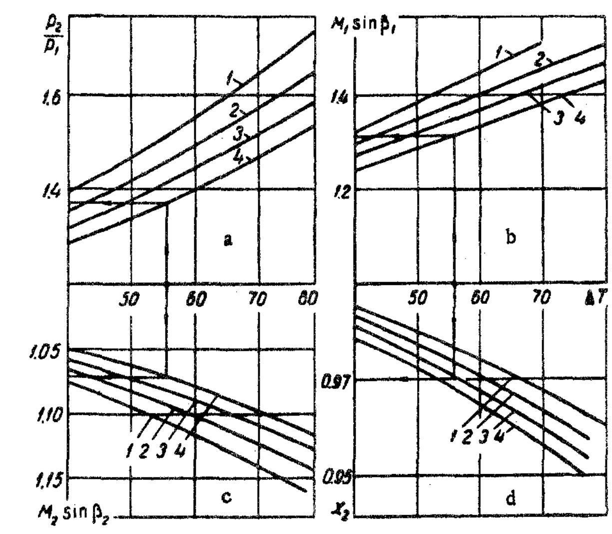

Fig. 2. Nomogram for calculating oblique shocks at low pressure: a) calculated p_2/p_1; b) $M_1 \sin\beta_1$; b) $M_2 \times \sin B_2$; d) calculated dryness of vapor behind shock; 1—$p_1 = 0.01 \times 10^5$ n/m^2; 2—0.05×10^5; 3—0.1×10^5; 4—0.5×10^5.

The order of obtaining the basic equation for calculating the compression shock is as follows.

From the momentum equation (3) we obtain

$$M_1 \sin\beta_1 = \frac{1}{2}\sqrt{\frac{T_1'}{T_1' - \Delta T}}\left[1 \pm \sqrt{1 + \frac{4(T_1' - \Delta T)}{kT_1'}\left(\frac{p_2}{p_1} - 1\right)}\right]. \quad (5)$$

The minus sign in front of the radical does not hold for supersonic velocities.

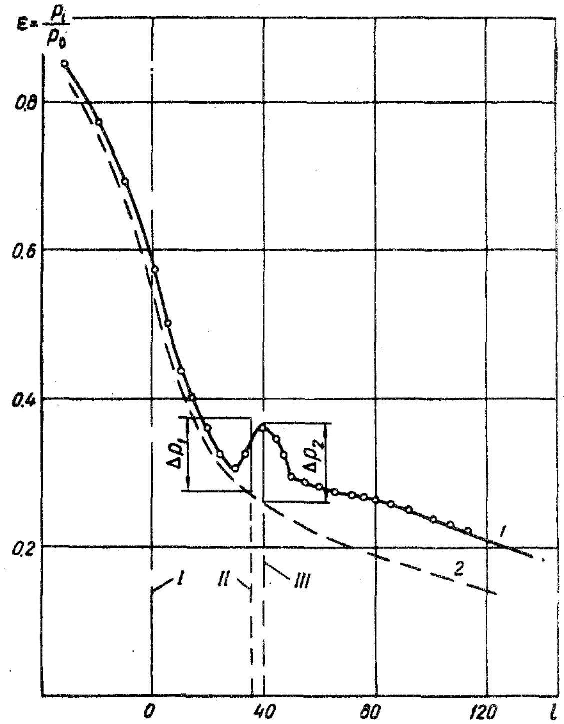

Fig. 3. Static pressure distribution along a Laval nozzle with $p_0 = 0.95 \times 10^5$ n/m^2 and $t_0 = 127°$ C: 1) experiment; 2) calculation (k = 1.3); I—throat; II—section at which shock occurs; III—section at which shock should occur according to the calculation illustrated in Fig. 1.

Taking the last relation into account, from the continuity equation and the energy equation, respectively, we obtain

$$x_2 = 2\left[1 + \sqrt{1 + \frac{4(T_1' - \Delta T)}{kT_1'}\left(\frac{p_2}{p_1} - 1\right)}\right]^{-1} \cdot \frac{T_1' - \Delta T}{T_1'}\,\frac{p_2}{p_1};$$

$$x_2 = 1 - \frac{c_p \Delta T}{r} + \frac{kRT_1'}{8r} \cdot \left[1 + \sqrt{1 + \frac{4(T_1' - \Delta T)}{kT_1'}\left(\frac{p_2}{p_1} - 1\right)}\right]^2 - \frac{kRT_1'}{2r}. \quad (6)$$

Therefore,

$$2\left[1+\sqrt{1+\frac{4(T_1'-\Delta T)}{kT_1'}\left(\frac{p_2}{p_1}-1\right)}\right]^{-1}\frac{T_1'-\Delta T}{T_1'}\frac{p_2}{p_1}=$$

$$=1-\frac{c_p\Delta T}{r}+\frac{kRT_1'}{8r}\cdot$$

$$\left[1+\sqrt{1+\frac{4(T_1'-\Delta T)}{kT_1'}\left(\frac{p_2}{p_1}-1\right)}\right]^2-\frac{kRT_1'}{2r}.$$

After simple transformations we finally have

$$\left[\sqrt{1+\frac{4(T_1'-\Delta T)}{kT_1'}\left(\frac{p_2}{p_1}-1\right)}-1\right]\frac{K}{2\left(1-\frac{p_1}{p_2}\right)}=$$

$$=1-\frac{c_p\Delta T}{r}+\frac{kRT_1'}{4r}\left[\frac{2(T_1'-\Delta T)}{kT_1'}\left(\frac{p_2}{p_1}-1\right)+\right.$$

$$\left.+\sqrt{1+\frac{4(T_1'-\Delta T)}{kT_1'}\left(\frac{p_2}{p_1}-1\right)}-1\right]. \quad (7)$$

For given values of p_1 and ΔT, the equation contains only one unknown p_2. Thus the intensity p_2/p_1 of the shock is determined by pressure p_1 and the supercooling of the flow. For convenience of calculation, Fig. 2 shows a nomogram for calculating compression shocks constructed on the basis of (7).

Thus, for given values of p_1 and ΔT, the ratio $p_2/$ $/p_1$ may be found from (7) or from the nomogram, and in turn the values of x_2 and $M_1 \sin\beta_1$ may be determined.

These values have been calculated from (5) and (6) and are shown in the second and fourth quadrants of the nomogram. The angle β_1 of inclination of the shock for a given value of M_1 is determined from the obvious relation

$$\sin\beta_1=\frac{1}{2M_1}\left[1+\sqrt{1+\frac{4(T_1'-\Delta T)}{kT_1'}\left(\frac{p_2}{p_1}-1\right)}\right]\cdot$$

$$\sqrt{\frac{T_1'}{T_1'-\Delta T}} \quad (8)$$

The entropy increase and the energy loss in the shock are determined on the basis of calculated values of p_2 and x with the help of an i-s diagram.

Figure 3 shows experimental data on the pressure distribution in the nozzle. The agreement between the experimental and calculated values is good. This is confirmed by a large body of experimental data obtained in the Department of Steam and Gas Turbines of the Moscow Power Engineering Institute [1, 3].

NOTATION

a) Velocity of sound; R) gas constant; T) temperature; p) pressure; k) isentropic exponent; ΔT) supercooling of flow; ΔT_{crit}) critical supercooling of vapor; x) degree of dryness; c) flow velocity; (β) angle of inclination of shock; τ) time for vapor to traverse nozzle segment between section at which it intersects the upper boundary curve and section at which the shock occurs; i) enthalpy; c_p) specific heat at constant pressure. Subscripts: 1 and 2 refer, respectively, to the state of the flow ahead of and behind the shock.

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